International Journal of Research in Advent Technology, Vol.3, No.2, February 2015
E-ISSN: 2321-9637

Design of Digital Internal Model controller for an Unstable CSTR Process

S. Nagammai[1], K.Dhanalakshmi[2], S. Latha[3]
Department of EIE[1,2], Department of EEE[3], KLNCE[1,2],TCE[3],India
snagammai731@gmail.com[1],srijaipriya@gmail.com[2], sleee@tce.edu[3]

Abstract- In industrial applications control of unstable process is more difficult than stable process. In addition the performance of the controller is restricted for unstable process because of the presence of right half plane poles or zeros in comparison with stable process. B.W.Bequette proposed a three state model for a continuous stirred tank reactor taking into account the cooling jacket temperature as a third state variable and which has high influence on the open loop and closed loop response. For an open loop unstable process it is suggested to use internal model controller (IMC) in conventional feedback form because IMC is internally unstable. Further it simplifies the tunning procedure because it needs only single tuning parameter. This paper deals with the design of digital IMC for a jacketed chemical reactor which is an open loop unstable system due to the effect of scale-up on the steady state and dynamic characteristics.The proposed digital controller is capable of providing system stability and also provides set point tracking and disturbance rejection. The controller is designed using the discrete transfer function for various values of filter factor. The simulation result shows the feasibility of using the proposed controller for control of the unstable CSTR process.

Index terms: IMC, unstable CSTR,

1.INTRODUCTION:

Industrial processes namely high purity distillation column, highly exothermic chemical reactor, pH neutralizer, batch and continuous reactors exhibit nonlinear behaviour. These processes may be required to operate over a wide range of conditions due to large changes in process inputs or set points. The classical two-state CSTR model is well-known to be capable of giving exotic behaviour. Russo and Bequette (1995) noted that, a three-state model (incorporating a jacket energy balance) could result in multiple steady-states whereas the two-state model exhibits a single steady state operating point. It is difficult to design conventional PID controller, when there is a significant change in process gain and when the process is open loop unstable. Temperature control of unstable CSTR process is generally crucial and complicated due to system nonlinearity.When conventional PID controllers are used to control highly nonlinear process, the controllers meed to be tuned very effectively in order to provide stable behaviour over the entire operating range. When there is a significant change in process gain and when there is open loop instability, the effectiveness of conventional PID controller become inadequate. Many researchers have developed controllers for SISO unstable CSTR process [3] &[5]. The internal model controller is the best suited method used to design controllers for unstable process. The paper is organized as follows: The description of unstable CSTR process is given in section 2. The details about design of discrete version of internal model controller(IMC) is given in section 3. Digital controller design for the unstable CSTR process is given in section 4. The simulated results are obtained and shown in section 5. The Conclusions of the work are drawn in section 6.

2. PROCESS DESCRIPTION

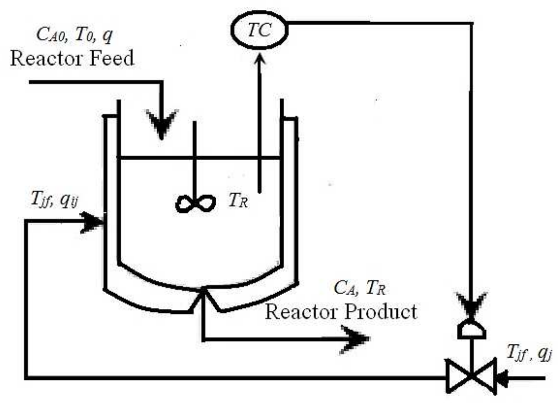

Fig.1. Schematic diagram of Jacketed CSTR process

A continuous stirred tank reactor (CSTR) whose schematic is shown in Fig. (1), is considered where in a first order exothermic reaction $A \rightarrow B$ takes place at a temperature T_R with a cooling jacket. The chemical reaction is first order with Arrhenius temperature dependence. In the jacketed CSTR the heat is either added or removed to compensate for the temperature difference between a cooling jacket fluid and the reactor fluid. The ordinary differential equations that model the CSTR behavior is given in equation [1]. The component material balance on the reactant gives

$$\dot{C}_A = f_1\left(C_A, T_R, T_j\right) = \frac{q}{V}\left(C_{A0} - C_A\right) - k_0\, e^{\frac{-E_a}{RT_R}} C_A \qquad (1.a)$$

Where 'q' is the feed flow rate of the reactant, C_{A0} is the feed concentration, C_A is the concentration of component A in the reactor, k_0 is the frequency factor, E_a is the activation energy, R is the ideal gas constant, T_R is the reactor temperature in degree Rankine. The energy balance in the reactor system is

$$\dot{T} = f_2\left(C_A, T_R, T_j\right) = \frac{q}{V}\left(T_0 - T_R\right) - \left(\frac{UA}{V\rho C_p}\right)\left(T_R - T_j\right) + \left(\frac{-\Delta H}{\rho C_p}\right) k_0\, e^{\frac{-E_a}{RT_R}} C_A \qquad (1.b)$$

Where $(-\Delta H)$ the heat of reaction, U is is the heat transfer coefficient, A is the heat transfer area, T_0 is the reactor feed temperature, T_j is the jacket temperature in degree Rankine. The energy balance in the jacket is

$$\dot{T}_j = f_3\left(C_A, T_R, T_j\right) = \frac{q_{jf}}{V_j}\left(T_{jf} - T_j\right) + \left(\frac{UA}{V_j \rho_j C_{pj}}\right)\left(T_R - T_j\right) \quad (1.c)$$

Where $\left(q_{jf}\right)$ is the jacket make up flow rate.

The variables C_{Ao}, T_0, q, q_{jf}, T_{jf} are all considered as inputs and out of which C_{Ao} and T_0 are considered as the disturbance variables. The manipulated variable is the reactor feed flow rate (q) and the controlled variable is the the reactor temperature.

The three nonlinear differential equations expressed in equations 1.a, 1.b and1.c cannot be solved analytically. The approximate model is derived about the steady-state operating point of the reactor.

The state space representation of the CSTR process in terms of deviation variables is given in equation [2]

$$\begin{bmatrix} \dot{C}_A' \\ \dot{T}_R' \\ \dot{T}_j' \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} C_A' \\ T_R' \\ T_j' \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix} \begin{bmatrix} q' \\ q_{jf}' \end{bmatrix} \quad (2.a)$$

The output state space model is,

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} C_A' \\ T_R' \\ T_j' \end{bmatrix} \qquad (2.b)$$

The output and input states are defined in the deviation variable form as,

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} C_A' - C_{AS} \\ T_R' - T_{RS} \\ T_j' - T_j \end{bmatrix} \quad u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} q' \\ q_{jf}' \\ C_{A0}' \\ T_0' \end{bmatrix}$$

The first two inputs are considered as the manipulated variables, while the last two are inputs are considered as disturbances.

The matricies A and B are,

Where, $k_s = \alpha e^{-\left(\frac{E_a}{RT_{RS}}\right)}$ & $k_s' = k_s\left(\frac{E_a}{RT_{RS}^2}\right)$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{22} \\ a_{21} & a_{22} & a_{22} \\ a_{31} & a_{32} & a_{22} \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} \end{bmatrix}$$

$$B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} \\ \frac{\partial f_3}{\partial u_1} & \frac{\partial f_3}{\partial u_2} \end{bmatrix}$$

Substituting the numerical values given in Table.1 &2, the constants in matrix A and B are evaluated and the state space model of the system thus obtained is given in equation (3).

$$\begin{bmatrix} \dot{C}_A' \\ \dot{T}_R' \\ \dot{T}_j' \end{bmatrix} = \begin{bmatrix} -7.99 & -0.014 & 0 \\ 2923 & 4.56 & 1.46 \\ 0 & 4.75 & -5.89 \end{bmatrix} \begin{bmatrix} C_A' \\ T_R' \\ T_j' \end{bmatrix} + \begin{bmatrix} 0.0007 & 0 \\ -0.48 & 0 \\ 0 & -3.26 \end{bmatrix} \begin{bmatrix} q' \\ q_{jf}' \end{bmatrix} \qquad (3)$$

Table1. CSTR variables and parameter values:

Variable	Description	Value
V	Reactor volume (ft^3)	85
V_j	Jacket volume (ft^3)	21.25
k_0	Arrhenius exponential factor (hr^{-1})	16.96 x 10^{12}
E	Activation energy (Btu / lb mol)	32400
U	Heat transmission coefficient (Btu/hr ft^2 °F)	75
A	Heat transfer surface area (ft^2)	88
R	Perfect gas constant (Btu /lb mol °R)	1.987
$(-\Delta H)$	Reaction Heat (Btu / lb mol)	39000
ρC_p	Product density X Thermal capacity (Btu/ ft^3 °F)	53.25

$\rho_j C_{pj}$	Water density X Water Thermal capacity (Btu/ ft^3 °F)	55.6

Table. 2. Reactor steady state parameter values

Variable	Description	Value
C_{Aos}	Steady state Feed concentration (lb mol / ft^3)	0.132
T_{0s}	Steady state Feed temperature (°F)	60
T_{RS}	Steady state Reactor temperature (°F)	101.1
T_{jfs}	Steady state cooling water input temperature (°F)	0
T_{js}	Steady state jacket temperature (°F)	80
C_{As}	Steady state Reactor concentration (lb mol / ft^3)	0.066
q_s	Steady state Feed flow rate (ft^3/hr)	340
q_{jfs}	Steady state cooling water flow rate (ft^3/hr)	24

The transfer function which relates the Reactor temperature (T_R) to the cooling water flow rate (q_{jf}) of the plant is obtained from state space model given in equation (3) and expressed as given in equation (4).

$$\frac{T_R(s)}{Q_{jf}(s)} = \frac{-4.753S - 38}{S^3 + 9.34S^2 + 16.98S - 34.2} \quad (4)$$

The transfer function indicates that the open loop system is unstable due to the presence of unstable pole at 1.1687 (RHP pole).

3.CONCEPTS OF IMC FOR UNSTABLE PLANT

In the chemical engineering field, the internal model controller is a popular technique and it is named so because the controller has model of the plant as its part. The IMC feedback configuration is shown in Fig.2.

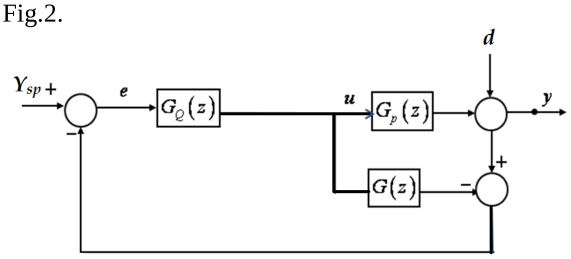

Fig.2. IMC feedback configuration

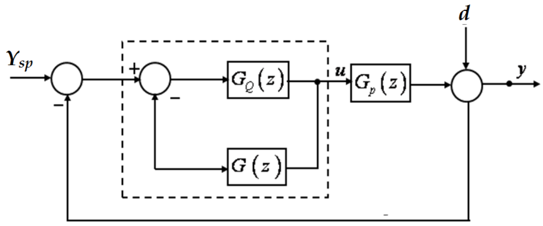

Fig.3. Standard control configuration with IMC

The actual transfer function of the plant is denoted as $G_p(z^{-1})$ and its model by $G(z^{-1})$. Let the factored form of model transfer function be represented as

$$G(z^{-1}) = z^{-k}\frac{B_g B^- B^{nm+}}{A} \quad (6)$$

Where,

B_g is the factor of B with the roots inside the unit circle and with positive real parts.

B^- is the factor of B that have roots with negative real part and which may lie either inside, outside or on the unit circle.

B^{nm+} refers to that part of B containing non minimum zeros of B with positive real parts.

The equivalence of standard control configuration with IMC is shown in Fig.3. In this schematic diagram, the block shown in dotted line is the controller in the conventional form (G_C) and is expressed as,

$$G_C = \frac{G_Q(Z)}{1 - G(Z)G_Q(Z)}$$

Suppose the plant $G(z^{-1})$ is containing one unstable pole. The internal stability of the system is assured if the following conditions are satisfied.

i) $G_Q(Z)$ is stable.

ii) At the unstable poles p_i of the plant $G(z^{-1})$, $(1 - G(Z)G_Q(Z))$ is zero.

That is, $$\left(1 - G(Z)G_Q(Z)\right)\Big|_{p_i} = 0 \quad (7)$$

It is achieved by introducing a parameter β in the definition of $G_Q(Z)$.

$$G_Q(Z) = G^{\dagger} G_f \left(1 + \beta z^{-1}\right) \quad (8)$$

Where,

$$G^{\dagger} = \frac{A}{B_g B_s^- B_r^{nm+}}$$

B_s^- is the steady state equivalent of factor of B^-

B_r^{nm+} is B^{nm+} with reversed coefficients.

Substituting the definition of $G_Q(Z)$ given in Equation (8) into Equation (7), we get

$$\left(1\text{-}GG^{\dagger}G_f\left(1+\beta z^{-1}\right)\right)\Big|_{z=p_i}=0 \qquad (9)$$

Solving equation (9), we obtain

$$\beta = p\left(\frac{1}{GG^{\dagger}G_f}\Bigg|_{z=p_i}-1\right) \qquad (10)$$

To account for the noise and model-mismatch , a low pass filter of the form given below is used.

$G_f \triangleq \frac{B_f}{A_f}=\frac{(1-\alpha)}{1-\alpha z^{-1}}$, where $1 > \alpha > 0$

The IMC equivalent conventional feedback controller is given by,

$$G_C = \frac{B_f A}{B_g\left(A_f B_s^- B_r^{nm+} - B_f B^- B_r^{nm+} z^{-k}\left(1+\beta z^{-1}\right)\right)} = \frac{S(z)}{R(z)} \qquad (11)$$

4.DIGITAL IMC DESIGN

Step1: The discrete version of the continuous time transfer function given in Equation (3) is indicated by $G(z^{-1})$

$$G(z^{-1}) = z^{-1}\frac{\left(-0.237\text{x}10^{-3} - 1.71\text{x}10^{-5}z^{-1} + 0.218\text{x}10^{-3}z^{-2}\right)}{1-2.91z^{-1}+2.82z^{-2}-0.911z^{-3}}$$

$$G(z^{-1}) = z^{-1}\frac{-0.237\text{x}10^{-3}\left[1+0.96z^{-1}\right]\left[1-0.95z^{-1}\right]}{1-2.91z^{-1}+2.82z^{-2}-0.911z^{-3}} = z^{-k}\frac{B(z)}{A(z)}$$

Comparing with equation (6), we find that

$$A = 1-2.91z^{-1}+2.82z^{-2}-0.911z^{-3}$$

$$B_g = -0.237\times10^{-3}\left[1-0.95z^{-1}\right]$$

$$B^- = \left[1+0.96z^{-1}\right] \text{ and } B^{nm+}=1$$

$$k_p = -0.237\times10^{-3}$$

Step2: The Q form of IMC is obtained using the formula,

$$G_Q(z) = \frac{A}{B_g B_s^- B_r^{nm+}}\frac{1-\alpha}{1-\alpha z^{-1}}\left(1+\beta z^{-1}\right)$$

It is evaluated that, $B_r^{nm+}=1$ and $B_s^- = 1.96$

The filter factor α is chosen as 0.1 and β is evaluated as 0.02 using equation (10).

The S and R polynomials required for the IMC equivalent of conventional feedback controller is obtained using equation (11) and given below.

$$R(z) = 10^{-3}\left(0.54+0.26z^{-1}+0.28z^{-2}\right)$$

$$S(z) = 0.9-2.6z^{-1}+2.5z^{-2}-0.77z^{-3}-0.016z^{-4}$$

5. SIMULATION RESULTS:

In order to analyze the performance, the proposed internal model controller shown in Fig.3 is simulated using MATLAB. The plant transfer function is discretized by using a zero order hold circuit. The controller term $\frac{S(z)}{R(z)}$ and the plant term $\frac{B(z)}{A(z)}$ are evaluated for simulation. The performance of internal model controller for various values of filter factor (alpha) is shown in Fig.4. for step change in reactor feed flow rate. It indicates that, as the filter factor increases the response exhibits offset. The robustness of the proposed controller is studied by applying a moving average random noise with the plant during simulation. The effectiveness of disturbance rejection capability is demonstrated by applying a step disturbance of magnitude 30 which begins at 5 hours and ends at 7 hours. The servo regulatory response of the proposed controller is shown in Fig.5.The performance comparison of designed controller is given in Table.3.

Table 3: Comparison of time domain Specifications

Parameter	IMC with α=0.1	IMC with α=0.6	IMC with α=0.8
Peak time t_p in hr	**0**	0.65	4.4
Rise time t_r in hr	**0**	0.04	0.08
Settling time t_s in hr	**0.02**	0.09	0.21
% peak overshoot	**0**	0	0

6. CONCLUSION:

Hence the digital internal model controller algorithm is demonstrated for a modeled unstable jacketed CSTR process. The performance summary given in Table.3 indicates that, the internal model controller designed with least value of filter factor offers better performance in terms of the time response specifications such as settling time, overshoot,peak time and rise time compared to higher values. The effectives of designed digital internal model controller is proved with the simulation results.

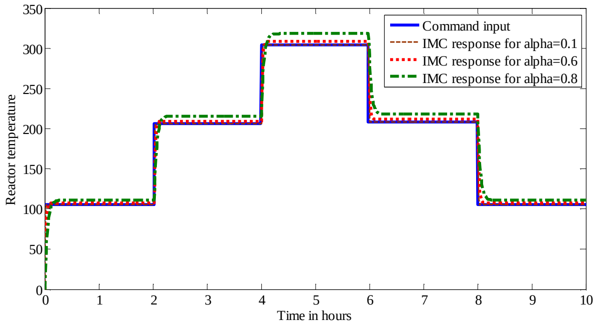

Fig.4. IMC servo response to set point change in coolant flow rate

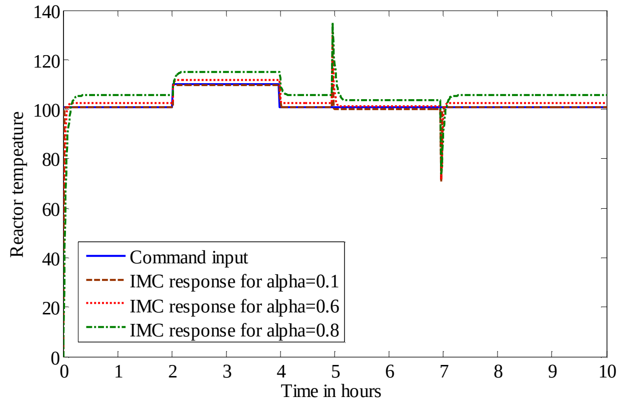

Fig.5. IMC Servo Regulatory response to step disturbance of magnitude 30 begins at 5 hours and ends at 7 hours.

REFERENCES:

[1]. A course on Digital Control, Kannan M. Moudgalya, IIT Bombay.

[2]. Bequette,B.W., Process control: modeling, design, and simulation ,Prentice Hall, New York(1998)

[3]. Ayadeva Bhat, M.Chidambaram & K.P.Madhavan, "Nonlinear Feedback Control of a CSTR", Chemical Engineering Communications , Volume 101, Issue 1, 1991

[4].Chidambaram M., Yugender Ch. "Model reference cascade control of nonlinear systems: application to an unstable CSTR", Chem Eng Comm. 1992; 101(1): 15–29p.

[5]. D. Krishna, K. Suryanarayana, G. Aparna, and R. Padma Sree "Tuning of PID Controllers for Unstable Continuous Stirred Tank Reactors" International Journal of Applied Science and Engineering, Volume 101, Issue 1, 2012.

[6].S. Nagammai, S.V. Prasanna Kumar, S. Latha, " Design of Digital Controller for An Unstable CSTR Process" Journal of Electronic Design Technology, Volume 5, Issue 3, December 2014